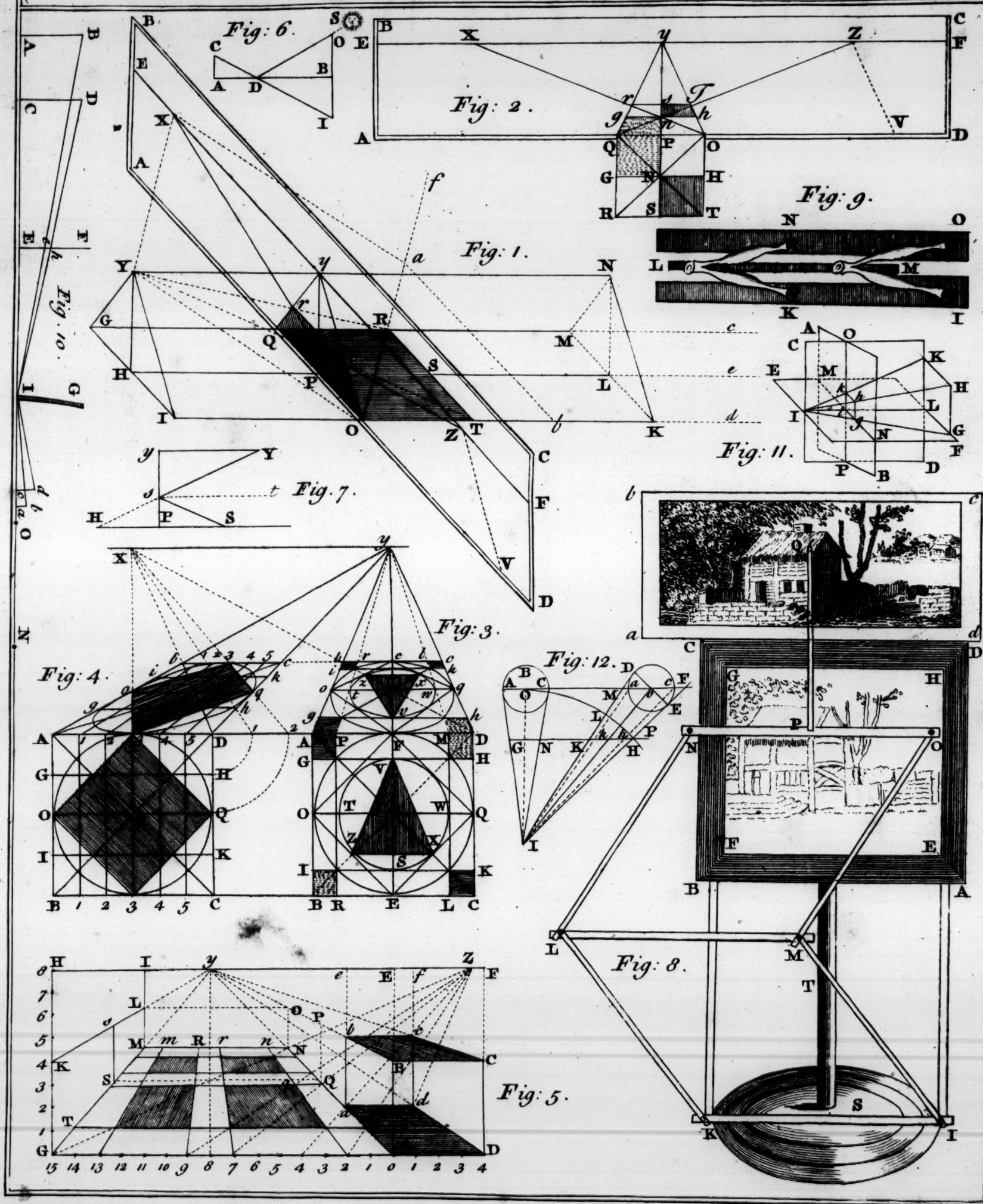


The THEORY & PRACTICE of PERSPECTIVE.



3

THE
PRINCIPLES
OF
PERSPECTIVE

EXPLAINED IN
A GENUINE THEORY;

AND APPLIED IN
An EXTENSIVE PRACTICE.

With the CONSTRUCTION and USES of all
such INSTRUMENTS as are subservient to
the PURPOSES of this SCIENCE.

By BENJ. MARTIN. *K*

L O N D O N :

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PRINCIPLES

PERSPECTIVE

ILLUSTRATED BY

A. G. H. THEORY



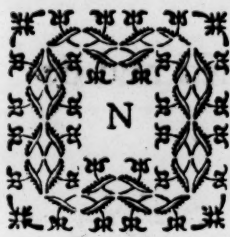
With the
such treatment
the British Museum

BY BENJ. MARTIN

LONDON



P R E F A C E.

 O Mathematical Science requires a THEORY more than PERSPECTIVE, and perhaps no one has been less attended with it in Publications of this Kind. The Manner in which some Authors have treated the Theoretical Part has not been satisfactory to me ; and therefore, as many others may be of my Way of thinking, I have presented them with a THEORY of PERSPECTIVE which appears to me the most genuine, natural, and perspicuous that possibly can be, as it consists of the fewest, and those the most simple Principles, *viz.* the Motion of a Point in a given Direction, and of a visual Ray, proceeding from it to the Eye at a given Distance, which Ray by its Intersection with the perspective Plane, marks out the perspective Appearance of the Point or Object upon it.

The Application of this Theory to Practice, in the common Way, is so easy and natural, that I think what is here said may suffice for that Purpose.

But as there are many important Particulars relative to the THEORY and PRACTICE of PERSPECTIVE not touched upon in Treatises of this Sort, I have added as much on that Head as I judged was necessary, to give the Student of the *Polite ARTS* as just a Notion of this Science as it deserves, and as he ought to conceive of it.

I have purposely avoided all Kind of Prolixity, and studied one Point only here, as in all my other Writings, *viz.* To make the Way to Science as easy and as concise as possible, and with the least Expence of Time and Money. The GREEK Proverb asserts, *A great Book is a great Evil*; the LATIN Adage says, *Verbum sat Sapienti*; and an ENGLISH Genius can see *Day at a little Hole*.

To conclude, this Work has been only the Recreation of a few leisure Intervals from Business, which I did not know better how to improve, in Regard to my own Amusement, and the Emolument of the Public; to which great Object, the Endeavours of every Individual of Society ought to be directed.

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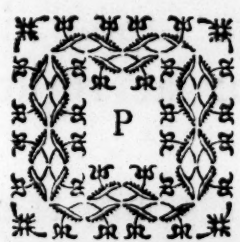
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CHAPTER I.

The Genuine THEORY of PERSPECTIVE demonstrated from its *First* PRINCIPLES.

 PERSPECTIVE is the Art of delineating upon a Plain Surface the Appearance of Objects, such as they have upon a Glass Plane interposed between them and the Eye, at a given Distance from that Plane.

For since the Sensation or Vision of any Point, is caused by a *Ray of Light* proceeding from that Point to the Optic Nerve (or *Retina*) in the Eye, and as this Ray passes through the transparent Plane in its Way to the Eye, the Point in which it transits the Plane is that which we call the *Place* or *Seat* of that Point upon the *Perspective Plane*.

But this Doctrine of the *perspective Seat* of a *Point*, a *Line*, a *Superficies*, and a *Solid*, must be illustrated by proper Diagrams, that it may become easy and perspicuous ; and that Method can only convey *clear Ideas*

B

that

that is in itself *natural*, *simple*, and consequently *genuine*. And such an one we shall here endeavour to pursue.

This Method consists in the Construction of a *Perspective* PRISM, and its Intersection by the *perspective* PLANE. The *Prism* is represented by YGHIKLMN; and the *Perspective Plane* by ABCD. The *Intersection* of the *Prism* and *Plane* is the Triangular Plane O γ Q, (See FIG. I.)

The Base of the Prism IGMK is *the Ground Plane*, in which all Objects in the *Plane of the Horizon* are considered, and their Perspective Seats determined in the Plane AC. The Vertex Y of the Prism is the Place or Position of the Eye at its given Distance Y γ from the Perspective Plane, which is here supposed to be posited directly before the Eye, or at right Angles to the Prism.

The Part of the Plane AD which stands on the Ground, is called the *Ground-Line*; the Point (γ) is called the *Point of Sight*; and the Line E γ F (parallel to AD) the *Horizontal Line*, for Reasons which will appear by-and-by.

Then in Order to determine the perspective Appearance of Objects upon the Ground or Horizontal Plane IGMK indefinitely continued out towards (*c d*), we must first conceive a Point to begin to move from the Plane AC along the indefinite right Line O *d*, which is perpendicular to the Ground Line A D; this Point is at the same Time in the Side IN of the Prism, and drawing the dotted Line YO, that will represent the *Visual Ray*, by which the said Point is seen by the Eye at Y.

Let

Let the Point in its Motion along the Line Od , arrive at the Situation T , and draw the Visual Ray TY by which it there appears. Then it is self-evident that the Visual Rays YO , YT , are in the Plane or Side of the Prism IN ; also that the Ray YT passes through the perspective Plane in the Point (t) , which therefore is the *perspective Seat* of the Point T .

As the Motion of the Point is continued, the Angle TYN constantly decreases; till at length the Point gets to an indefinite Distance, and then that Angle vanishes, and the Ray YT coincides with the Line YN , and the Object, now in the *Horizon*, has its perspective Seat on the Plane AC in $(y.)$

By this indefinite Motion from the perspective Plane in O to the Horizon, the Visual Ray YT keeps constantly moving in the Side or Plane of the Prism IN , and of Course it must trace out by its Passage through the perspective Plane the Line Oy , which is the common Intersection of the two Planes IN and AC ; and therefore the said Line Oy is the *perspective Seat* or Appearance of the Line or Side Od of the Ground Plane indefinitely extended.

In like Manner if a Point moves from Q along the Line Qc indefinitely extended, the visual Ray will describe the Line Qy on the perspective Plane, in which it intersects the Side GN of the Prism: Also another Point moving from P in the Line Pe to an indefinite Distance, the Ray proceeding from it will describe upon the Plane AC the Line Py , which is the common Section of that Plane with the vertical Plane of the Prism HN .

Hence

Hence it is again self-evident, that if a Line QO were to move from the Ground Line to an indefinite Distance, in a Direction Od , and always parallel to itself, or to the Ground Line, it would appear on the perspective Plane to describe the Triangle OyQ , which therefore will be the Perspective of that indefinitely extended horizontal Plane $dOQc$, described by such a Motion.

Hence also it is manifest, that any Line QO , however large at the perspective Plane, yet removed to an indefinite Distance, *viz.* to the *Horizon*, will there subtend no sensible Angle, but vanish in a Point at (y) . Therefore all Objects in the Horizon will appear in the Line EF passing through the Point (y) , which is therefore called the *horizontal Line*.

All indefinite right Lines perpendicular to the Ground Line of the Plane, as Od , Pe , Qc , have their *Perspective* expressed by right Lines converging to the Point of Sight (y) , as Oy , Py , Qy , which are called *Radial Lines*.

The *Perspective* of any determinate Space $QOTR$, is thus found on the Plane; t and r being the *Seats* of the Points T and R on the Ground Plane, Ot and Qr will be the Perspectives of the Sides OT and QR ; and drawing rt , that must be the perspective of the Side RT ; and so the *Trapezium* $QOtr$ will be the perspective of the proposed Area $OQRT$.

If OT be equal to QR , then is Ot equal to Qr , and tr is parallel to OQ . If $OT = QO$, then is the Area $OTRQ$ a Square, and $OtrQ$ the Perspective of that Square.

Let

Let the Diagonal OR of the said Square be continued out indefinitely towards f ; then upon the horizontal Line EF, take y X and y Z equal to y Y, the Distance of the Eye from the perspective Plane; and draw XY. Also make $O b = OI (= Y y,)$ and draw $b a$ parallel to OQ or RT; and $O b a$ being a right Angle, $O a$ will be the Diagonal of a Square, and equal and parallel to YX.

Then if a Point be now conceived to move along this Diagonal Line $O f$, to an indefinite Distance, the *Visual Ray* will contain an Angle with the Line YX that will constantly decrease, and at last vanish, when the Point is in the Horizon; and at last, by a Coincidence of the Ray with the Line YX, the said Point in the Horizon will have its perspective Seat at X in the horizontal Line EF.

During this Motion of the Point in the Diagonal of a Square, the Visual Ray will trace out the Line OX in the perspective Plane. Wherefore, because this Line $O f$, and all others parallel to it, converge to the Point of Distance X upon the Plane, they are called *Diagonal Lines*.

Hence any indefinite Line drawn from the Point O, between OT and OR, will have its perspective Seat in some Part between Oy and OX, and every such Line between OR and OQ will have its Seat in the same horizontal Line beyond X; and such Points in the Line EF to which these Lines tend are called *accidental Points*.

Hence the Line X y between the Point of Sight y, and the Point of Distance X, is the Perspective of 45 Degrees

Degrees in the Horizon ; or the Double thereof X Z is the Perspective of 90 Degrees in the Horizon.

Hence also it is evident that since the Original Lines Qc and Of cross each other in R, therefore the Seat of the Point R will, in the perspective Plane, be at (r) where the Radial Qy is intersected by the Diagonal OX. And from thence the following general Rule is derived for finding the perspective Seat of any given Point or Object upon the Ground Plane, *viz.*

GENERAL RULE.

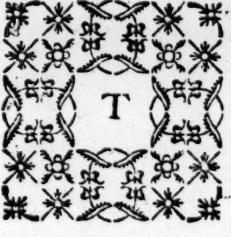
From the given Point, as T, draw a Perpendicular to the Ground Line as T O ; from the Point O draw the radial O y ; and from the same Point O set off the Distance OT from O to Q in the Ground Line, and draw the Diagonal QZ, and where it crosses the Radial, as at (t) , there will be the Seat of the Point T required.





CHAP. II.

A DEMONSTRATION of the foregoing general
RULE of *Practical* PERSPECTIVE, in
regard both to *direct* and *oblique* VIEWS.

 THE THEORY of PERSPECTIVE required a Position of the *perspective Plane* ABCD. (FIG. 1.) with an Elevation above the *Ground Plane*, which in that Figure (as in most real Cases) is at right Angles to it. But the practical Rule we have thence deduced, admits a *Delineation* in *Perspective* of any Sort of Objects with the utmost Truth and Facility by placing the *Perspective Plane* upon the *Ground* or *Horizontal Plane*.

Thus (FIG. 2.) let ABCD be the transparent or perspective Plane in an horizontal Position, and let T be the given Point whose perspective Seat is to be found on the Plane AC in this Position; and then by the Rule the Process is the same here, as before in the other Figure. For drawing the Perpendicular TO to the Ground-Line AB, and from O, the Radial Oy to the *Point of Sight* (y); and then making QO = OT, and drawing the Diagonal QZ to the Point of Distance

tance Z it will intersect the Radial Oy in the Point (T) the *perspective Seat* of the Point T as required. Therefore also OT will be the Perspective of the Line OT .

To demonstrate the Truth of this Method, it will be necessary to make the following Construction in the Diagrams, viz. Take OV in the Ground Line (in both Figures) equal to the Distance of the Eye OI ($= yZ$), and draw ZV , which will be parallel to the Radial YO . Then (in FIG. 1.) there are two similar Triangles formed, viz. ITY , OTt ; and VQZ , OQT (in FIG. 2.)

From the first we have this Analogy, $IT : OT :: IY : Ot$; and from the second, this, $QV : QO :: ZV : OT$. Now the three first Terms in each Analogy are respectively equal to each other, that is $IT = QV$, $OT = OQ$, and $IY (= Oy) = ZV$. Therefore the fourth Term Ot is the same in the first with OT in the second; consequently the Intersection of the Radial Oy and Diagonal QZ is the Seat of the Point T as true by the *practical Method*, in FIG. 2. as by the THEORY in FIG. 1.

If therefore $OTRQ$ be the Square before-mentioned, its Perspective will be $OTrQ$ as determined by the Theory, and OR and QT will be seated in Or and QT ; the Center N of the Square in (n), and the Sides QR and RT in Qr and rT .

From what we have said it will be easy to conceive how the Perspective of any Figure may be put into Perspective by inscribing it in a Square, and in the Perspective of that Square marking the Seat of all the Points in the extreme Parts or Angles

gles of the given Figure; and then, lastly, by connecting these Points with right Lines, the Perspective of the given Figure is formed.

Thus for Example (in FIG. 3.) let ABCD be the Square upon the Ground Plane, placed directly before the Eye at (*y*); in this Square are inscribed the following Figures, (1.) The Square EOFQ with an Angle F in Front; (2.) A Circle circumscribing that Square, (3.) A Square inscribed in this Circle in a Situation similar to the original Square; (4.) Another inscribed Circle; and in that, (5.) an isosceles Triangle ZVX. (6.) The small Squares IR, &c. in the Angles of the original Square.

Then it is plain, by Inspection only, how all the Seats of the Points O, I, B, R, E, &c. are found in the perspective Square *A^bcD*; and by properly connecting them, how all the Perspectives of the several Figures result; and even Circles are easily traced with an even and dextrous Hand through so many given Points. These Figures are readily seen and distinguished by the corresponding small Letters in each.

The Forms and Shapes of Figures in their perspective Plan are greatly altered in a *direct View*, but still more so in one that is *oblique*, as is evident in (FIG. 4.) where the same Square is placed on one Side the Point of Sight (*y*); and its Perspective *A^bcD*, determined by the Radials, *Ay*, *Dy*, and the Diagonal *DX*, contains the Perspective of the Circle and Square in Front very different in their Forms, Dimensions, &c. from what they had before in the direct View.

C

If

If the original Square be divided by equidistant Lines perpendicular to the Ground Line AD, as 11, 22, 33, &c. their Perspectives all converge to the Point of Sight (γ) and the perspective Line bc will be equally divided by them. Any Lines GH, OQ, IK parallel to the Ground Line by transferring their Distances to the Ground Line, (that is, making $D_1 = DH$, $D_2 = DQ$, &c.) and then drawing the Diagonals X_1 , X_2 , &c. the Points b , q , k , c , will be given in the Radial $D\gamma$ through which to draw their Perspectives gb , oq , ik , bc , in this *oblique View*, the same as in the *direct one*.

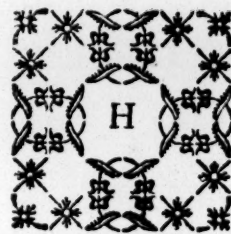
Though Right Lines perpendicular or *oblique* to the Ground Line differ in their Dimensions in the direct and oblique Views, yet those right Lines that are *parallel* to the Ground Line, are the same in both Views. For in the *oblique View* we have $AD : bc :: \gamma D : \gamma c$; and in the *direct View*, it is $AD : bc :: \gamma A : \gamma b$; But in the former γD is to γc in the same Ratio as γA to γb in the latter; therefore the Ratio of AD to bc is the same in both; consequently, if AD be the same in both, then will bc be so too.





C H A P. III.

The PRINCIPLES of *Scenographic* PERSPECTIVE, explained and demonstrated.

AVING demonstrated the perspective Principles of ICHNOGRAPHY, or all that is concerned in the *Ground Plan*; we have, in Effect, done the same with respect to the SCENOGRAPHY, or all that relates to *Elevations*, or *Upright Structures*, built upon the said Ground Plan.

For whether any Right Line as AD (FIG. 5.) be upon the Ground, or elevated above it, as AB, yet if those Lines are at an equal Distance *Ay* from the Point of Sight, their perspective Diminutions will be equal; and they equally result from, and may be determined by, the same *General Rule* above demonstrated. For draw the Radials *Ay* and *Dy*, and the Diagonal AZ, which will cross the Radial *Dy* in (*d*); and draw *ad* parallel to AD, then will this Line *ad* be the Perspective of a Line equal to AD, and parallel to it, and at the same Distance from it. Consequently *AadD* will be the Perspective of a Square upon the Ground Plane whose Side is AD.

In like Manner upon the Point D erect the Perpendicular $DC = AD$, and compleat the Square $ABCD$; then draw the Radials By , Cy , and the Diagonal BZ cutting Cy in (c) , and draw bc parallel to BC ; so shall $BbcC$ be the Perspective of the same Square (*viz.* equal to $ABCD$) placed upon a Ground Plane parallel to AD , and elevated above it to the Height of AB ; and at same Distance from it.

By joining the Points a , b , and c , d , 'tis evident, the whole Figure becomes the Perspective of a Cube standing on the Ground Line AD , whose Side in Front (or that next the Eye) is $ABCD$; the Base $AadD$; the Top $BbcC$; the Right Side $AabB$; the left Side $DdcC$; and the farthest or hinder Side $abcd$.

What remains is to shew that the Perspective ab of a given Line in an Elevation above the Ground Plane is equal to the Perspective of it ad , when it coincides with the same. And this is easily seen in the similar Triangles AyD , ayd , and AyB , ayb . For from them we have these Ratios $AD : ad :: Ay : ay :: AB : ab$; But $AB = AD$, therefore $ab = ad$. And this holds good in very Elevation of the Line AB , since the Angle BAD is not concerned in the above Analogies.

If the Sides of the Cube be continued up to the Line of Sight yZ , there will be formed an equal sided *Parallelopiped* upon the same Square Base $AadD$, but whose upper Surface vanishes into a right Line $eEfF$. The Front View of this Solid is $AEFD$; the Side $AaeE$ is called the *Profile* or Side-View, which is greater or less in Proportion to the Obliquity of the View; for *in front*, or a *direct View* it vanishes, as is easy to observe from the Reason of the Thing in FIG. 3, and 4.

If the Parallelopiped were carried to a Height above the Line of Sight equal to CF, and was hollow or transparent, then would the under Part of the Top, or highest Surface, have its Perspective exactly equal to BbcC; and if it extended equally above and below the Line yZ, then the Perspective of the Top will be the same with that of the Bottom, viz. AadD.

When it is considered that by having the Distance, Altitude, Angle of Position, and Profile of any Object given, the Perspective Seat of every Point, and therefore of every Line, will be easily formed by the foregoing practical Rule, it will be needless to multiply Words to shew how the Perspective of any Object may be compleatly found by delineating those of the Points and Lines which compose it, as they lie in the Perspective Sides of the Cube or Parallelopiped which circumscribe the Object; As the *Point of Sight* and the *Point of Distance* are always known, and also the *Accidental Points* from the given Position of the Object with respect to the Ground Line, there can remain no Difficulty to the ingenious Artist.

And hence, that which has been hitherto considered as a mighty Difficulty in Perspective, viz. *drawing the Perspective of Objects by Reflection, as those seen in Water, &c.* will be found none at all. For if they are Things in the Heavens, their Altitude measured by a Quadrant gives their Elevation above the horizontal Line HZ; and since, by the Laws of CATOPTICS the *Angle of the incident Ray is ever equal to that of the reflected one*, therefore the Point of the Image is in a Right Line drawn from the Object perpendicular to the Horizon.

rizon HZ, and at the same Distance below as the Object itself is above it. Hence the Place of the SUN, CLOUDS, TOPS of MOUNTAINS, &c. in *Day-Pieces*; and the MOON, STARS, &c. in *Night Views*, may be readily and truly drawn in Perspective, by the same *General Rule*.

But if the Object OB be upon the Ground, (FIG. 6.) and its Height, and Distance AB be known, then divide the Distance AB, in the Point D, so that it may be $BD : AD :: OB : AC =$ to the Height of the Eye at C; also make $IB = OB$, and draw CI; then will the Angle $CDA = IDB = ODB$; and consequently, if Water be at D, the Image at I will be seen by the Eye at C. Or a General Rule for finding AD is $\frac{AB \times AC}{OB + AC} = AD$.

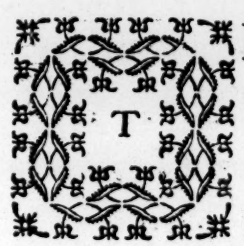
But more generally and concisely by *Gunter's Lines* proceed thus; let the Altitude of any Body, on the Earth or in the Heavens, be denoted by the Angle ADC, which suppose to be 30 Degrees, and let the Height of the Eye AC be 6 Feet. Then if you extend the Compasses from 30° to 45° in the *Line of Tangents*, that Extent will reach from 6 Feet to 10,4 Feet in the *Line of Numbers*, which will be equal to AD, the Distance of the Point D in the Plane of Reflection (or Azimuth of the *Phænomenon*) from the Eye.

The Distance AD being found in the *Ground Plane*, its *Seat* on the *Perspective Plane*, or *Picture*, will be found by the Intersection of a *Radial* and *Diagonal*, as above taught.



CHAP. IV.

The ANALYSIS of PERSPECTIVE; or the METHOD of *analysing* a given *Perspective* VIEW or LANDSCAPE.

 HE ANALYSIS of PERSPECTIVE consists in the Resolution of a *Perspective Picture* into its *Prototype* or original Component Parts, so that we may form a just Idea of their Situations, Distances, Dimensions, &c. from their Delineation in a given *Landscape*, or Perspective PICTURE.

The first Step to be taken in this Process is to construct two LINES of MEASURES, one for taking the Dimensions, Distances, &c. of Objects upon the *Ground Plane*, and the other for those which have an Elevation above it. The first of these is to be laid upon the Ground-Line (as AG FIG. 5.) and is nothing more than the Division of the lowest Line of the Landscape or Picture into *equal Parts*, which may represent *Feet*, *Yards*, *Rods*, or *Miles*, as the Nature and Extent of the Perspective require.

The second *Line of Measures* for Uprights, is to be affixed to the Perpendicular Line GH on the Side of the

the Landscape. By the Line of Measures on the Ground Line AG, the Length of any Perspective Line parallel thereto may be instantly known; as also the Height of any Line perpendicular to the Perspective Plane or Landscape, in any Part, may be estimated by the Line of Measures at the Side.

In this Operation we have regard to the Point of Sight (y); and this in any Landscape is easily found by continuing out any two extreme Radial Lines, as AN and GM, till they meet, which will be in the Point (y) required. In the Diagram (FIG. 5.) the Scale of Measure upon the Ground Line AG contains 15 equal Parts, *Feet, Yards, &c.* and the perpendicular Scale GH has eight. And therefore if Lines were drawn by the Edge of a Rule from the Point (y) to each of those Divisions, they would shew in any Part of the Ground Plane, or Side Plane, the proper Length of any perspective Lines parallel to the said Scales, and of Course the Form of *Walks, Canals, Vistas, &c.* which are perpendicular to the said Scales.

Thus for Example, suppose a Ruler laid from (y) to 2, and by its Edge you draw the Line $n 2$; then will the Space A N $n 2$ be the Perspective of a Walk or Vista of 2 Yards in Width, by the Side of the *Perspective Garden*. And for the same Reason G M $m 13$ will be such another on the other Side; and 7 r R 9 will be the same through the Middle of the Area.

But now to resolve and measure any *Parallelogram* in Perspective parallel to the Ground Line, will require first a Determination of the Point of Distance Z; for by *Diagonal Lines* drawn from the Divisions of the

the Scale AG to that Point, the extreme Line AN in the Side of the Landscape is divided, and by these Divisions the perspective Distances of Objects between any two Parallels to the front Line AG are estimated, or become known.

Thus if the Diagonal Z *z* be drawn, it will cross the Line AN in *t*, and drawing through *t* the Parallel T *t*, it will make the perspective Walk TA of 2 Yards Width, contiguous to the Ground Line. Again, if Diagonals from Z to 7 and 9 be drawn, they give two Points in the Radial AN at Q, through which two Parallels are drawn, which form the same Walk QS, through the Midst of the Garden. And in like Manner such a Walk is determined at the farthest Side MN.

But it is evident, the *nearer* Z is to *y*, the greater will be the Divisions A *t*, AQ, AN; and less, as it is farther removed. Therefore it must be determined and fixed for the given Landscape, which is not always very easy to be done. But no Difficulty can arise in such perspective Pieces as have any *regular Figure* in a Side View upon the Ground Line. Thus the two *Summer Houses* in the perspective View of CHELSEA-HOSPITAL and GARDENS being of equal Sides, afford a perspective Square, as A *a d D* (FIG. 5.) the Diagonals A *d* determines the Point of Distance Z. Also the *Octagon Bason* in the Court before *Buckingham House*, (now the *Queen's Palace*) will shew the same Point in the View of St. *James's Park*.

To analyze a Building or any other Object in any Part of the Landscape, you are to observe, that all the *Ichnography* of it is resolved and measured by the Method

thod just mentioned, and the *Scenography* is determined by measuring the upright Lines thus: Let the Parallels be drawn for the angular Points, and where they intersect the Side Radial, as suppose at S, there erect perpendiculars equal to the Heights on those Angles of the Building as S s, then the Radial y K drawn through s will give the Measure GK of the true or real Height of the Building in that Part; and thus it will be found for any other.

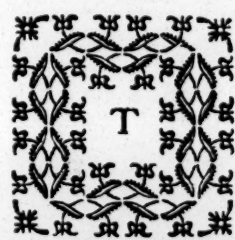
Hence the Height of *Trees, Houses, Men, &c.* are easily found on any Part of the Landscape, and the Manner or Ratio of their Diminution GK, S s, ML, as they are removed from the perspective Plane to the farthest Part of the View. Hence also it appears that Objects below the Eye rise constantly in the perspective View towards the *horizontal Line* HZ. And those *above* the Eye in the same Manner *descend* to it, as they go more remote from the Plane. Thus *Birds* diminish to our View, and apparently descend towards the Horizon, when they fly from us in a Direction parallel thereto.

C H A P.



CHAP. V.

The MECHANISM of PERSPECTIVE; or
Mechanical CONSTRUCTION of the THEO-
RY thereof, for a DEMONSTRATION to the
SIGHT.

 HE MECHANISM of PERSPECTIVE is the
next Thing necessary for the young Designer
to be acquainted with, and to have often be-
fore his Eyes. It consists in constructing
or building up the THEORY of PERSPECTIVE into a *real*
or *material* FORM, that the *Rationale* of the Science
may, as it were, become the Object of Sight, and be
thereby much easier conceived by the Mind.

In order to this it will be necessary to have a rectan-
gular Piece of Wood (*Mahogany* or *Ebony*) about 24
Inches long and 6 or 8 wide; in this at about 10 In-
ches from the End, is to be made a Dove-tail Groove
half an Inch deep and wide, and then a Frame must be
made to move or slide freely in the said Grove, yet so
as to be always upright or perpendicular to the Sur-
face on which it stands. This Basis or long Parallelo-

D 2

gram

gram of Wood will then answer to the general Ground-Plane GHIKLM (in FIG. 1.) The Frame to ABCD, the perspective Plane; and the Groove to the Ground Line OQ.

At or near the End must be placed a Stile or Wire represented by the Perpendicular HY, with a small Hole in the upper Part at Y, also another is to be placed at the other End denoted by LN, with a Perforation at N.

In the Frame AC must be fixed a large Piece of transparent Horn, such as is used for *Lanthorns*. Then having a Square made of a thin Piece of Brass represented by QOTR, with fine Holes in the Angles and Sides at Q, P, O, R, S, T, and the other Parts when necessary; the Perspective of this Square is to be drawn upon the *Plate of Horn* with a sharp Steel Point, in the Manner shewn in FIG. 2.

This being done, through the several Points in the Perspective Square as at Q, P, O, T, s, r, g, h, and y, let small Holes be drilled, and then the Horn being fixed in the Frame AC, that must be so placed in the Groove QO that the *Ground Lines* of both the *Original and Perspective Plane* may coincide as near as possible.

Then for the *Visual Rays*, they will be very properly represented by *fine Threads* fixed in and passing through the Holes in the Stiles and Planes, &c. Thus if upon the Board GK, be drawn the parallel Lines GM, IK, upon Paper, Pasteboard, Ivory, &c. as also the Lines GI and MK: And also if Threads fixed in G and M be put through the Holes in the Stiles at Y and N, and then carried down and fixed in the Points I and K;

K; and, lastly, if another Thread pass from Y through the Point (γ) in the Horn, and be fixed in the Hole at N; then will the *perspective PRISM*, be properly constructed.

If the Line OR be continued out upon the Board to (a) and a Thread be fixed in that Point, and from thence carried through a Hole in the Point of Distance X in the Horn to the Point Y in the Stile, and there fixed, you will have the Plane OaXY constructed for the Demonstration of what relates to the Perspective of *Diagonal Lines* and their *Parallels*.

Lastly, let Threads be fixed in the Holes of the Square OQRT, and carried through the Holes in the Plate of Horn, and all of them fixed tight in the Hole Y of the Stile HY, then will the Square *Pencil of Visual Rays* be properly formed for shewing the Reason of every Thing which concerns the Perspective of *Radial Lines*, or those which are *perpendicular* and *parallel* to the *Ground Line*; especially if to these are added other Threads from Holes in the Line OT continued out, and passing through corresponding Holes in the Horn which will determine the Perspectives or Landscapes of any given *Parallelograms*.

All that has been hitherto said relates to the *Mechanical Construction* of the THEORY; but the *Practical* Part of PERSPECTIVE may also be proved and confirmed by Experiments made in the same Machine with a little Variation; for in this Case there will be no Need of Threads at all; and instead of a Stile HY in Form of a Wire, it should have a round Plate

Plate of Brass on the Top, with a small Hole in the Middle to look through, like a common *Sight-Vane* to Quadrants, &c. Then if several Pieces of Horn (very smooth and transparent) were fitted to the Frame AC, so as to be taken in and out at pleasure, and upon these several Perspectives of Object drawn, as of a *Chair, Table, Inside of a Room, Doors open, &c.* Then if these are successively, and appositely placed by the Ground Plane of the Original, and viewed through the Sight Vane at Y, the *Prototype* and its *Perspective* will be seen perfectly to coincide upon the Plate of Horn in every Point and Part; and consequently the *Truth of the Draught* is established.

In this last Experiment instead of *Horn*, a Plate of *polished* GLASS would do much better, as no Holes thro' it are necessary, were it not for two Objections; first the Lines or Landscapes are not so easy to draw upon Glass, unless by an accustomed and skilful Hand; and secondly, when well drawn, are liable to be broke and destroyed. And, indeed, some Plates of Horn may be found but little inferior to Glass, and will therefore render it less necessary.

However, it will be proper here to observe, That the Perspective of Objects by *Reflection* is the same with that by *Refraction*, as will appear from the Reason of the Thing; for, let SPH be the Ground Line (FIG. 7.) P y the Section of the Perspective Plane; S the Place of an Object, from which a Ray S s falling upon the Plane in (s) is reflected to the Eye at (y), and continuing y s to H, the Point H will be the apparent Place of that Object, and PS will be equal to PH, by the com-
mon

mon Principle of Optics *that the Angle of Incidence Sst is equal to the Angle of Reflection tsy* ; But $Sst = sSP$, and $tsy = sHP$; therefore the Effect, or Sensation in the Eye, is the same whether it be seen by *Reflection from S*, or by *Refraction from H*, and the Perspective Seat (s) is the same upon the Plane in both Cases.

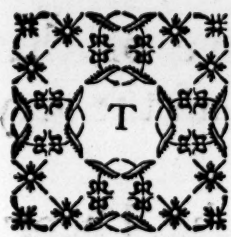
Therefore if (in FIG. 2.) ABCD represent a plain LOOKING-GLASS (such as is usually placed over the Chimney-Piece in a Parlour) then all Objects exposed to it are viewed in *true Perspective* upon it. And if the Line of Sight EF, Height of the Eye Py , and Points of Distance X, Z, were drawn upon it with Ink, or black Varnish, and OQRS a Square laid horizontally before it, then drawing the Radials Oy , Qy , and the Diagonals OX, QZ, you will have the Perspective of that Square, *viz.* $QOTr$ upon the Face of the Glass the very same as by Refraction in FIG. 1.

It will be very easy to understand that the Theory of Perspective may be constructed for a perspective Plane that has any given Inclination to the Ground Plane; since the Distance of the Eye from the Plane is, in this Case, *equal to the perpendicular Distance from the Ground Line and Co-tangent of the Plane's Inclination added together, the Radius being the Height of the Eye*; and all other Parts of the Process will be the same, or similar to those above described.



C H A P. VI.

The CONSTRUCTION and USE of a *Perspective* TABLE, for readily drawing in true PERSPECTIVE any natural SCENE, or VIEW of OBJECTS proposed.



THE Method of putting any Objects into a Perspective View by practical Rules derived from the Theory, as above delivered and explained, requires the Knowledge of their various Dimensions, and Distances, and is a Work of Time, with many other Circumstances, the Consideration of all which together has made it necessary to contrive still more practical and expedient Methods of delineating any given Object, or Groupe of Objects, in true Perspective.

And this has been found as easy and practicable by the Invention of a *Perspective* PLAIN TABLE, as can be desired; and of these Tables, there have been many Forms and Contrivances since that first invented by Sir. C. WREN. They differ only in Form and Construction, the Principle of the Operation is the same in them all. That which I have
here

here given in a Diagram admits, I think of the greatest Facility and Certainty of Motion, which is the utmost Excellency of this Sort of Instruments.

This *Perspective drawing* TABLE is represented in FIG. 8. It may consist of a Board, with a Rectangle or Frame ABCD, to take off and put on, upon Occasion, whose Use is to hold firmly a smooth Sheet of Paper EFGH upon the Body or Plane of the Table in the same Manner as in the *Plain Table* for SURVEYING. But this *perspective Table* may be made entire, without such a Frame, with proper Springs on the Sides to hold the Paper for the Draught.

This Table placed upon a Foot and Pillar ST, may be put into a *vertical Position*; it is not necessary it should be perfectly perpendicular, and therefore requires no Plumb-Line or Spirit-Level.

The Table being fixed pretty steady, you next apply, or fix to it, a Set of three moveable Quadrangles ABIK, IKLM, LMNO, which have all a perfect Freedom of Motion at their respective Angles I, K, L, M, N, O; These are connected with the lower Part of the Table by the Ends A, B, going into proper Grooves or Holes with Spring-Catches, for holding them very fast. In order to make them as light as possible, it will be sufficient to have the whole Mahogany, except the four Sides IM, KL, LN, MO, which may be small Slips of Brass.

When Things are thus disposed, as in the Diagram they appear, it is plain the Part NO of the upper Quadrangle, is freely moveable over all the Plain of the Table, and therefore in the Middle at P is

E

screwed

screwed fast on, a Stile or perpendicular Piece PQ, with a Point at Q called the *Index* of the Stile or *Tracer*.

This Tracer PQ has in its lower Part at P a Pencil fixed in the screw, by which it is fastened to the Piece or Bar NO. Now let $(abcd)$ be an imaginary transparent Plane, as in the Theory (FIG. 1. and 2.) upon which let us suppose the *Perspective* of a *House*, *Trees*, *Walls*, &c. to be delineated; and that this Plane is placed upon the upper Part of the Table CD.

Then it is evident, if the Hand holding the Pencil at P, were to move in such Manner as that the Index Q might be made to pass along or describe any Line on the perspective Part, the Pencil itself must necessarily describe a Line of the same Length, and in the same Position, upon the Paper on the Table; and consequently, by this easy Artifice, all the *Contours* or Out-Lines of the *Landscape* will be readily delineated upon the Paper of the *Perspective Table*.

Again it is evident, that the Eye placed at such a Height and Distance from the Plane $(abcd)$ as belong to the Landscape drawn upon it, will see all the Lineaments of Objects in the said Picture coincide exactly with those of their *Prototypes* or *Originals*, by the visual Rays passing to the Eye, from corresponding Points in each; Therefore it will follow, that if the said perspective Table $(abcd)$ were removed, and a *Sight-Vane* fixed in the proper *Point of Sight*, an Eye looking through it, would perceive the Index

Q

Q pass over the same Lines in the original Objects, as before in the Landscape, if moved in the same Manner; and consequently the Pencil at P will describe the same Lines as before; and thus a true *perspective Draught* upon the Table ABCD will be made of any distant Objects proposed.

This Practice is universal for all Objects remote or near, for the *Sight Vane* being moveable, may be fixed at any Distance from the Table, and Height above the Horizon, and consequently may be thereby adapted to any Distance of Objects, or Size of the *Drawing* proposed; for from a bare View of FIG. 2. it is evident that the *perspective Plane* QO *tr* or *Landscape* will be larger as the Height of the Eye P *y* is greater, at the same Distance *y* X.

Also it is as evident, that for the same Height of the Eye *y* P, the *Landscape* will increase or decrease as the Distance of the Eye from the *perspective Plane* decreases or increases.

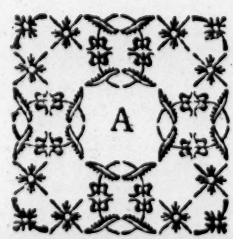
Hence when the Eye is at a great Height, and the Distance of the Plane or Table from it very small, the Perspective of a given Square will be *enormously great*, and almost of a *triangular Form*. Therefore a *Face*, *Head*, &c. drawn in the Square will be greatly *deformed* when delineated on the *perspective Draught* of the Table. But this very deformed Image will appear to an Eye at that given Height and Distance as *formous* and perfect as the Original. And thus the whole Doctrine of the *Anamorphosis* or *Deformation of Images* is elucidated, as it were, by Inspection.

The DOCTRINE of SHADOWS is reckoned a PART of PERSPECTIVE ; but it is rather only the *reverse* of the direct Method ; for if (in FIG. 1.) we suppose a CANDLE placed at Y, at the Distance HP from the perspective Plane AC, and if OQ *rt* represent an *Opake Object* upon that Plane, then it is evident that Rays of Light proceeding from the *Candle*, will project the SHADOW of that Object into a perfect *square Form* OQRT on the *Ground Plane* GK. So that *no new* PRINCIPLES are required for a further Prosecution of this Subject ; and a superfluous *Copia Verborum* is not necessary to the ingenious young Student in this Art.



C H A P. VII.

The CONSTRUCTION and USE of a *Double Parallel RULER* in PERSPECTIVE, ARCHITECTURE, &c.



NO Instrument is of so great and general Use in all Kinds of Drawing in Architecture, Perspective, &c. as the PARALLEL-RULE, I shall here give the Construction and Description of one which is quite New, and better accommodated to perspective Drawing and Designs in parti-

particular, than those of the common make. This may be properly called the *Double Parallel Ruler*.

The Construction of this Ruler is immediately derived from the Mechanism and Contexture of the two Quadrangles IKLM, and LMNO; and is in reality nothing else, excepting that the Parts or Bars IK and NO are of double the Length, continued out at the Ends K, and N; and the Middle or small Bar LM made longer at the End M. All this is evident in the Form of the Ruler itself (FIG. 9.) a little Way opened, where the same Letters denote the same Parts in both Figures 8 and 9.

The Design of this Parallel-Rule is to command the whole Extent of the perspective Table without being moved; that is, one Side of the said Rule IK (FIG. 9.) being fixed by two *steady Pins* to the lower Part of the Frame EF, the other Bar or Side of the Ruler ON is capable of being opened parallel-wise over the whole Table, or quite to the Top GH; or from the Side FG to the Side EH, at one Opening.

This Opening of the Ruler is not only *parallel*, but *perpendicular* likewise; or the Ends of the moveable Bar are always perpendicular over the Ends of the fixed one. This evidently follows from its being a *double Ruler*, or consisting of two single ones connected, *viz.* IKLM and LMNO (FIG. 8.) For as much as the End L over-runs the End K in the first, just so much must the End O go out beyond M in the second; consequently the Points O and N will be vertical to I and K in the whole Operation.

From

From this Construction of the Double Parallel and Perpendicular Ruler, it appears that the Extent of the Opening will be equal to twice the Length of the Side-Pieces IM or MO (FIG. 8.) that is, nearly the Length of the Ruler itself as represented in FIG. 9.

Hence it is plain, that by this Contrivance all Lines in the perspective Drawing on the Table which are parallel to the Sides, may be examined, continued, and compleated without the least Trouble, and with the greatest Certainty and Truth. Also all other Lines in the Drawing are commanded at one opening and placing the Ruler; and so all the constituent Lines being drawn, the whole Perspective or Landscape is finished and compleated in much less Time, and with greater Ease and Truth, than it could be by the *Common Parallel-Rules*.

The PRINCIPLE, therefore, and *Rationale* of the PERSPECTIVE TABLE and Double PARALLEL-RULER are the same in each; and it would be more prolix than difficult to shew how very useful such a *plain* DRAWING TABLE and PARALLEL RULER would be in all Branches of practical Mathematics, and particularly in drawing the various ORDERS of ARCHITECTURE, EDIFICES, and DESIGNS of all Kinds.

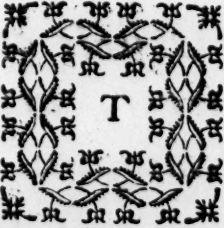
I shall conclude with one Observation more, and that is, that since the End O of the moveable Bar NO always slides along the Side EH of the Table, it is evident that if a *Line of Inches* (or any other *equal Parts*) were placed upon the Side EH, and a proper *Nonius* upon the End of the Ruler at O, that then *parallel Lines* might be drawn at any given Distance from each other,

other, to the 100th or 400th Part of an Inch, if required. Hence there will be no need to expatiate on the extreme Utility of the Ruler thus improved in drawing the *parallel Lines at their requisite Distances*, which constitute the several Members and Ornaments of PILLARS, PILASTERS, and their PEDESTALS, &c. in every Order, and according to the Measures proposed by the most celebrated *ancient or modern ARCHITECTS*.



C H A P. VIII.

A Demonstration that PERSPECTIVE is only a Branch of the SCIENCE of OPTICS; and that a PERSPECTIVE PICTURE is the same THING as a *Landscape* formed in the Focus of Optical Glasses, whether MIRRORS or LENSES.

H A T PERSPECTIVE is only one particular Case of OPTICS is evident to every one who understands the Principles of that Science. But for the Satisfaction of such as are not Proficients in the Mathematical Part of Optics, I shall here subjoin an easy and concise Demonstration of the
above.

above Position, deduced from the well known Principles of OPTICS.

In order to this, let IG (FIG. 10) be a *convex* SPECULUM, exposed to a given Object AB, at a given Distance IA, and let N be the Center, O the geometrical Focus, and IN the Radius of the said Mirror; lastly, let (*ab*) be the Image of the Object AB formed by the Mirror; *then their Proportion will always be that of their Distances from the Glass, viz.* $AB : ab :: AI : Ia$.

Again it is demonstrated by Writers on Optics, that $IA \times IN = Ia \times \overline{2IA + IN}$; Hence by this Equation and the foregoing Analogy, we get $AB \times IN = ab \times \overline{2AI + IN}$, which gives this Analogy, *viz.* $AB : ab :: 2IA + IN : IN$.

In like Manner if CD be any other Object, at the Distance IC from the Mirror, and (*cd*) be its Image, then it will be $CD \times IN = cd \times \overline{2ID + IN}$; If the Objects AB and CD are equal, then we have $ab \times \overline{2AI + IN} = cd \times \overline{2CI + IN}$; and from thence, this Analogy, $ab : cd :: 2CI + IN : 2AI + IN$.

Now when Objects are at a great Distance, the Radius of the Speculum will be inconsiderable, and may therefore be neglected in the last mentioned Analogy, which will then become $ab : cd :: 2IC : 2IA :: IC : IA$; that is, *The Images of all equal Objects at different, but great Distances from the Mirror, are inversely as those Distances, in their linear Dimensions.**

And

* Or more concisely thus; $AB : ab :: AI : Ia$; and $CD : cd :: IC : Ic$; therefore when $AB = CD$, we have $\frac{ab \times AI}{Ia} = \frac{cd \times IC}{Ic}$; but at great Distances, it is $Ia = Ic$, nearly; then $ab \times AI = cd \times IC$; whence $ab : cd :: IC : IA$, as above.

And this is the very Proportion in which the *Perspectives* of all Objects encrease or decrease upon the perspective Table or Plane, as is thus easily shewn. Instead of the Glass, let the *Point of Sight* or Place of the Eye be at I, viewing the Objects AB, CD, through the perspective Plane EF; and by drawing the Visual Rays IB, ID, they will define the Perspectives of these Objects upon the Plane, *viz.* Ee, Eb; Then because in the similar Triangles IBA, and IeE we have $AB : Ee :: IA : IE$; and the similar Triangles ICD and IEb, give $CD : Eb :: IC : IE$; consequently if $AB = CD$, we have $Ee : Eb :: IC : IA$; that is, *the perspective Appearances of equal Objects are inversely as their Distances from the perspective Table*; and therefore, *they are perfectly similar to those Images which are formed of distant Objects by optical Glasses.*

What has been now demonstrated for a *convex Speculum*, is shewn in the same Manner for a *concave One*; as also for a *convex Lens* of any Kind; wherefore it is very evident, *that the whole Science of Perspective is only one particular Case of Optics, viz. that where the Picture of Objects, at a great Distance, is represented to the Eye by the Glass.*

From this View of Perspective, several curious and interesting Problems will admit of a Solution in this Science, which do not occur in Treatises on this Subject; for it will be easy from what has been premised, to determine in what Case the Image or Landscape on the perspective Table, is equal to that in the convex Mirror or Lens. Because, in this Case, we have

$$\frac{AB \times IE}{IA} = Ee \text{ in Perspective; and in the convex Mir-}$$

F

ror

ror it is $\frac{AB \times IN}{2AI + IN} = ab$; therefore since, by Supposition, $Ee = ab$, we have $\frac{IE}{IA} = \frac{IN}{2AI + IN}$; consequently $IA + IN = IE \times \frac{2AI + IN}{IN}$.

From this Equation three different Problems will arise, *viz.* (1.) Given the Radius of the Mirror IN and Distance of the Object IA , to find the *Distance of the perspective Plane* IE , which will be $IE = \frac{IA \times IN}{2IA + IN}$. (2.)

Given the Distance of the Object, and of the perspective Plane, to find IN the *Radius* of the Mirror, we have $IN = \frac{2IA \times IE}{IA - IE}$. (3.) Given the Radius of the Glass, and Distance of the perspective Plane, to find the *Distance of the Object*, which will be $IA = \frac{IE \times IN}{IN - 2IE}$.

Hence also it appears, that when $IN = 2IE$, then IA will be infinite, or that when the Distance of the perspective Table is equal to half the Radius of the convex Mirror, *then the Pictures or perspective Appearances of Distant Objects must be the same in each*, as shewed before.

After the same Manner Theorems are deduced for the Solution of the same Problems with a convex Lens, whose Radius, or focal Distance, is IN ; for in this Case we have $IA \times IN = IE \times \overline{IA - IN}$; and therefore any two of the three Quantities IA , IE , IN , being given, the other is known from the Equation.

From what has been demonstrated, the great and excellent Use of *convex* MIRRORS and LENSES to all *Drawers, Designers, Painters, &c.* must evidently appear; as they exhibit Pictures of Objects in all their native and possible Perfection, and indeed far beyond the
Power

Power of Art or Genius of Man to equal; but still this is no Reason why they should not be copied and imitated as *Patterns* and *Originals*; As all artificial Drawings, though by the most exquisite Pencils, are nothing but Resemblances of some particular Scenes of Nature supposed to be formed by these Glasses. But there is something peculiar in the Nature and Effect of each, which may be useful to the young Draughts-Man to know.

The *convex* SPECULUM presents to the Eye a View of the most perfect and Genuine LANDSCAPE of all distant Objects to which it is exposed; this Picture is always *erect*, and therefore in a fit Position to be copied; all Objects are here projected according to the most perfect Laws of Perspective; the *Contours* or Out-lines, *Light* and *Shadow*, and all other Picturesque Incidents, are here what the young Designer should endeavour to impress his Mind and Memory with, in the strongest Manner, as being the Standard of all Perfection in his Art; and the only Pattern for forming just Ideas of every Part thereof. For it will be a Maxim that will ever hold true, *that Painters will always excel in Proportion as they imitate Nature more truly.*

The *convex* LENS is likewise of great Use to Designers, as it presents them with a *real Picture* or *Landscape* of Objects, and not one that is merely *ideal* or *picturesque*, as in the foregoing Case. This Landscape by the *Lens* is actually formed upon the Paper before your Eye, and may be readily copied or traced out by the Hand of the Artist; and for this Purpose the portable *Camera Obscura* is the most valuable Instrument, as it immediately furnishes an original Picture of any proposed Scene or Group of natural Objects

ready for the Hand to imitate and copy into a Landscape. From all which it appears that both Art and Nature have abundantly contributed to render the *Art of PAINTING* as perfect as possible; if the Professors of that Art be negligent in the Use of such Means, that is another Case, though but too common.

From these Premises it likewise follows, that the young Artist may have it in his Power *to draw any Object in any Proportion less than the Life*; for suppose the given Proportion of the Object to its Image be that of m to 1, then from the above Theorem for the *convex MIRROR* we have $\frac{m-1}{2} \times IN = IA$; and in the *convex Lens*, we have $\frac{m+1}{2} \times IN = IA$; Therefore by knowing the Radius IN of the Mirror or Lens, you have the Distance IA , at which the Glass must be removed from the Object, so that its Image may be less in the Proportion of 1 to m .

For Example, suppose it be required to draw an Object four times less than $m = 4$; and suppose the Radius of the Mirror $IN = 24$ Inches, $\frac{m-1}{2} \times 24 = \frac{3}{2} \times 24 = 36$ Inches, the Distance IA at which the Object must be placed from such a Mirror that its Image may be just a *fourth Part* as long and as broad.

But in Case of a Lens whose Radius (or focal Distance) is $IN = 24$, as before, you have $\frac{m+1}{2} \times IN = 5 \times 24 = 120$ Inches, the Distance of the Object from the Lens, that it may be four times less.

If the Image is required equal to the Object, then $m = 1$; therefore $IA = 0$, or the Object must touch the convex Mirror, because in that Case $\frac{m-1}{2} = 0$. But

in

in the Lens we have $\frac{m}{m+1} = 2$, which shews that $2\text{ IN} = \text{IA}$, or that the Object must be placed at twice the focal Distance of the Lens to have an Image equal to itself. And hence the excellent Use of the PROPORTIONAL CAMERA OBSCURA, contrived for readily producing an Image in any given Proportion less than the Object.

We have not yet mentioned the *concave* MIRROR, tho' it must be allowed to have the greatest Variety of curious Effects of any Optic Glass whatever. Among others, *this will present to the Eye an Image of an Object, not only in any Degree less, but also larger than the Life.* The Theorem here being $\frac{m+1}{2} \times \text{IN} = \text{IA}$, where 'tis evident, that when the Image is required to be made equal to the Object, or $m = 1$, then $\text{IA} = \text{IN}$, or the Distance of the Object and its Image both coincide with the *Radius of Concavity*.

The principal Use of a *concave* MIRROR, is in reversing a Landscape, or any perspective View, that is in giving a *Relievo* to the Picture, or setting every Thing upright, or in its natural Position. This it must necessarily do, because the Effect of a *concave Mirror* is just contrary to that of a *convex one*; and therefore as the *convex Mirror* projects all upright Objects in *Plano*, in a Picturesque Landscape, so the *concave Mirror* will reverse that Picture, and raise the several Objects from the Plane to their natural upright Position.

Hence a perspective View of the CITY of LONDON (for Instance,) being delineated from a Picture thereof in a *convex Mirror*, would when placed before a *concave Mirror* of the same Radius, present to the Eye a
delight-

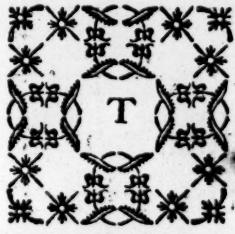
delightful Scene of the City, and all its CHURCHES, TOWERS, BUILDINGS, &c. erect, the same as it appears to the naked Eye at the *same Distance and Height above it* as the convex Glas was placed in.

It is no Wonder that the Effects of optical and perspective Glas are so naturally pleasing and delightful to every ingenious Mind, when it is considered, that the very Principles upon which they depend are founded in absolute HARMONY itself; for the original Equation above mentioned, *viz.* $IA \times IN = Ia \times \frac{IA + IN}{2}$ gives this Analogy, $AN : IA :: aN : Ia$; and therefore the Line AN is *always divided into musical or harmonical Proportion, in the Points A, I, a, N*, that is, *by the Place of the Object, the Place of the Mirror, the Place of the Image, and the Center of the Speculum, whether convex or concave*, I thought it was necessary the young Artist should be acquainted with so noble a Principle in the Theory of his Science, especially as all Authors of Perspective have been silent on this Head.



CHAP. IX.

The PRINCIPLES of *Spherical* PERSPECTIVE demonstrated, and applied universally to the *Geographical* and *Astronomical* PROJECTIONS of the SPHERE in *Plano*, for the *Construction* of MAPS, PLANISPHERES, ANALLEMMA, &c.

 HE PRINCIPLES of *spherical* PERSPECTIVE are next to be explained; and from thence it will appear, that they are the true Basis of all the Variety of PROJECTIONS of the SPHERE in *Plano*, that are so common in astronomical and geographical Treatises; and not only so, but it will be manifest that the Doctrine of *Spherical Projections*, which treated in the common Way is very intricate and confined, will, by the RULES of PERSPECTIVE, be rendered facile and universal.

To this End it will be necessary to consider the Construction of a Figure consisting of *three Planes* (as FIG. 11.) of which one is the *perspective Plane*, or *Plane of Projection* AB. The second is CD in a vertical Position to the Horizon, and is therefore called the *vertical Plane*. The third is the Plane EF, which passes through the Eye at I, and being parallel to the Horizon

zon is called the *horizontal Plane*. These Planes are all supposed to intersect each other at right Angles; Whence the Intersection MN is called the *horizontal Line*; and OP the *vertical Line*. And, lastly, the Line IL is called the *Principal Ray*, as coinciding with the Axis of the Eye.

Let H be a given Point, whose perspective or Seat (*b*) is to be determined upon the perspective Plane AB; from the Point H let fall the Perpendiculars HK to the vertical Plane, and HG to the horizontal Plane; and draw the Lines IK, IH, IG, which will pass thro' the Plane of Projection in the Points *k*, *b*, *g*. Then the Position of the Point *b*, in the Plane of Projection AB, with respect to the vertical and horizontal Lines OP, and MN, is thus determined.

The Planes being all given in Position, their intersecting Lines MN, OP, are given in Position on the Plane of Projection. Also the Position of the Line IL is given; and because the Position of the Object H is given, its Distances HK from the vertical Plane, and HG from the horizontal Plane are known; also its Distance *IL* from the perspective Plane, as likewise that of the Eye *I*, are both known. Then in the similar Triangles ILK and *I**l**k* we have $IL : I l :: IK : I k$. And the similar Triangles IKH, *I**k**b*, give $IK : I k :: HK : b k$; consequently we have from both Analogies, $IL (= I l + l L) : I l :: HK : b k$; which in Words is

GENERAL RULE I.

As the Sum of the Distances of the Eye and the Object from the Plane of Projection is to the Distance of the Eye

Eye from that Plane, So is the Distance of the Object from the vertical Plane, to the Distance of its Seat or Picture from the vertical Line.

And in like Manner, we have $IL : Il :: HG : hg$; which gives in Words

GENERAL RULE II.

As the Sum of the Distances of the Eye and Object from the Plane of Projection, is to the Distance between the Eye and this Plane, so is the Distance of the Object from the horizontal Plane, to the Distance of its Picture from the horizontal Line.

It is manifest that if the Object be placed between the Eye and the perspective Plane, these Proportions will be the same, only using the Word *Difference* instead of *Sum* in the first Term of each.

To apply what has been premised, it is evident that in Projections of the Sphere, the above mentioned three Planes may be applied, but they will be of a *circular Form*; and then by these Rules, any Point upon the Surface of the Globe or Sphere will have its Seat ascertained in the Plane of Projection, for any Position or Distance of the Eye whatever. But all Distances in this Case must be estimated in Parts of the *Radius* to a *Table of Sines*, as will be easy to understand by the following Examples.

Let it be required to find the Perspective or Seat of any Point upon the Surface of the Globe projected upon the PLANE of the EQUATOR, as seen by the Eye at a given Distance in the Axis of the Globe produced.

G

The

The three Planes are in this Example, (1.) AB the *Plane of Projection*, or that of the *Equator*. (2.) CD the *vertical Plane*, which is that of the *Equinoctial Colure*. (3.) EF the *horizontal Plane*, which is that of the *solstitial Colure*. For these three Planes are all at right Angles to each other.

The Place of the Star upon the Globe being given by a Table of Right Ascensions and Declination, we shall have (by the *Doctrine of the Sphere*,) this Analogy, *viz.*

As the Radius is to the Sine of Right Ascension, so is the Co-sine of Declination to the Star's Distance from the vertical Plane.

And then again,

As Radius to the Co-sine of Right Ascension, so is the Co-sine of Declination to the Star's Distance from the horizontal Plane.

The Distance of the Star from the Plane of Projection, is the *Sine of its Declination*, and the Distance of the Eye from the said Plane is to be expressed by some Multiple of Radius, as nR ; the Value of (n) being taken at Pleasure, and $R = 10000 = \text{Radius}$.

Thus let the Place of ALDEBARAN or the *Bull's Eye* be required in a PLANISPHERE upon the PLANE of the EQUATOR; The *Declination* of this Star is 16 Degrees North; the Sine of which is 2756, therefore that is the Distance of *Aldebaran* from the *Plane of Projection*.

The

The Right Ascension of this Star is $65^{\circ} 30'$, therefore
say, As Radidus ——— ——— 10,

To the Sine of Right Ascension $65^{\circ} 30'$ 9,959023

So is Cofine of Declination, 74 0 9,982842

To the Star's Distance from the *ver-*
tical Plane ——— } 8747 — 9,941865

Then again, As Radius ——— ——— 10,

To the Cofine of Right Ascension $24^{\circ} 30'$ 9,617727

So is Cofine of Declination, 74 0 9,982842

To the Distance of the Star from
the *horizontal Plane* — } 3986,2 — 9,600569

Having the Distance of the Star from the three
Planes, and the Distance of the Eye $nR = 10000n$.

You say (by General Rule I.) As $10000n + 2756$

: $10000n$:: $8747 : \frac{n \times 87470000}{10000n + 2756} =$ Distance of *Alde-*
baran from the vertical Line in the Planisphere.

Then (by Gen. Rule II) say

As $10000n + 2756 : 10000n :: 3986,2 :$

$\frac{n \times 39862000}{10000n + 2756} =$ the Distance of the Star from the hori-
zontal Line in the Projection.

Thus you have the Place or Seat of the Star in the
Planisphere for any Distance of the Eye whatsoever.

If we put $n = 1$, then is the Eye upon the Surface of
the Globe, *viz.* in the *South Pole* of the World, and

the *Distance of Aldebaran* from the *vertical Line* will be

$\frac{87470000}{12756} = 6857,3$ of such Parts as the Radius of the

Planisphere contains 10000.

Again, the Distance of the Star from the *horizontal Line* is, in this Case, $\frac{39862000}{12756} = 3125$. And thus the Places of all the Stars are put down in a Celestial Planisphere according to the common *Stereographic Projection* upon the *Plane of the Equator*.

The Stars may in the same Manner be projected upon the PLANE of the ECLIPTIC, and thus from the PRINCIPLES of PERSPECTIVE, you see the Reason and Foundation of those two excellent celestial Projections of the Constellations upon the Planes of the ECLIPTIC and EQUATOR published some Years ago by Mr. SENEX.

If the Surface of the Terrestrial Globe were to be thus projected upon the Plane of the Equator, the Places must be put down by the same Rules, but instead of *Declination* you take the *Latitude* of the Place; and for *Right Ascension* you take the *Longitude* from the *first Meridian*, or *vertical Plane*. And by this Means you will have the old *Ptolomaic MAP of the WORLD*, in the Center of which is the *North Pole*; the *Meridians* are all *right Lines*; and the *Parallels of Latitude* are all *concentric Circles*.

The common MAPS of the WORLD, are Projections upon the Plane of the *first Meridian* taken at Pleasure; the vertical Circle is the *Plane of the Meridian* at 90 Degrees from it; and the Horizontal Circle is the *Plane of the Equator*; and in the Intersection of the two last, the Eye is placed at the Distance *n.R.*, as before.

Thus for Example, let the MERIDIAN of LONDON be the Plane of Projection, and let it be required to find
the

the Seat of the City of CONSTANTINOPLE in the MAP; the Latitude of this City is 41° , and Longitude East 29° . Then the Distance from the *horizontal Plane* is the *Sine* of the *Latitude*; and since the Difference of Longitude between this City and the vertical Plane is 61° , you say,

As Radius to the Sine of the Difference of Longitude, so is the Cosine of the Latitude to the Distance of Constantinople from the vertical Plane.

And again,

As Radius to the Cosine of the Difference of Longitude, so is the Cosine of the Latitude to the Distance of the City from the Plane of Projection.

Having thus found the Distances from the three Planes, you have the Distance of *Constantinople* from the *vertical Line* by the first General Rule of Perspective; and the Distance from the *horizontal Line* by the second; and therefore you have its true Place in the *Plane of Projection*.

If we put $n = 1$ as before, then the Eye is upon the Surface of the Globe, *viz.* in the Pole of the opposite Hemisphere; and the MAP becomes that of the vulgar *Stereographic Projection*.

If the Place be in the Equator, its Distance from the horizontal Plane is $= 0$; and its Distance from the vertical Plane is the Sine of the Difference of Longitude, which put $= s$; and the Cosine thereof $= C$; then by Rule I, we have $nR + c : nR :: s : D = \text{Distance from the vertical Line}$, in the horizontal Line of the Map, and when $n = 1$, it will be $\frac{Rs}{R+c} = D$. Let $s = \text{Sine}$

s = Sine of 10 Degrees, then $D = 8749$. But if s = Sine of 80°, then $D = 8391,4$ which taken from Radius = 10000, leaves 1608,6; therefore the Width of 10° of the Equator in the Middle of the Map will be to the Width of 10° at the Side, as 875 to 1608 $\frac{1}{2}$. Consequently it appears how much *such Maps* must differ from the Truth of GLOBES.

Let the Radius, or half the horizontal Line, be divided into a Number of equal Parts, denoted by (p), and let m denote any Number of them; then will $\frac{mR}{p} = \frac{nR_s}{nR + c}$, whence we have $n = \frac{mc}{sp - mR}$. Hence if $p = 90$, and $m = 10, 20, 30, \&c.$ successively, you will have the Distance of the Eye, *viz.* nR given for projecting all the Meridians at an equal Distance upon the *Map*, and which therefore may be entitled the GLOBULAR PROJECTION, or MAP OF THE WORLD.

For Example; suppose $m = 10$, and $p = 90$, then $m : p :: 1 : 9$, therefore $n = \frac{c}{sp - R} = 1,7496$; and therefore $nR = 17496$, the Distance of the Eye for projecting the Meridian of 10 Degrees, and so for the rest.

If we suppose the Eye at an *infinite Distance*, then all the Meridians are projected (by the *parallel visual Rays*) into *Ellipses*; and the Distance of any Star from the *horizontal Line* will be the *Sine of its Declination*; and Radius will be to the Sine of Right Ascension, as the Cosine thereof is to the *Distance of the Star from the vertical Line* in the Projection. This Projection is of little Use in *Geography*, but of very great

great Use in *Astronomical Sciences*, and is known by the Names of ANALEMMA, and *Orthographical Projection* of the SPHERE.

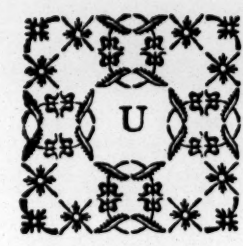
There are many other Projections of the Sphere, &c. to which the Rules of Perspective may be applied, but such Applications I shall leave to exercise the Ingenuity of my young Reader; it must suffice that I have here given him the Cue.





APPENDIX.

Concerning the Difference between *Optical*
and *Perspective* PLANES; and of the
APPEARANCES of OBJECTS upon them.

PON Consideration of that vulgar Error
of representing a COLONADE of PILLARS in
a *Front View*, all of an EQUAL SIZE, being
found in all the voluminous Authors on
Perspective, both Ancient and Modern, I was willing
to precaution the young Artist against it, as it is contra-
ry to the Nature of Perspective, and a Disgrace to the
Professors of the Science; some of whom, to my
Knowledge, have yet been so ignorant of its Principles,
that they have not scrupled to defend so absurd a
Position. And indeed if you say, that PILLARS in any
View must have their *Perspectives* unequal, but those
of PILASTERS must in a front View be all *equal*, you find
so few that can assign (or, indeed, see) the Reason of
it, as too plainly shews how little the very first Rudi-
ments of Perspective are understood by People in
general.

I mention the first Rudiments, because such Mistake is evidently owing to not attending to the Difference between the Appearance of Objects upon the *Optical*, and the *perspective Plane*, on which they are viewed; nor do they advert to the Difference that will arise in the perspective Appearance of Objects upon the same Plane, upon Account of their different Number of Dimensions; all which Considerations were thought *too elementary* to be allowed a Place even in the Beginning of a Treatise of Perspective, and are therefore reserved for an *Appendix* only.

This Affair may be thus represented and explained; Let O be the Center of a Circle ABC, placed in the Ground Right Line OF, *directly* before the Eye at I (see FIG. 12.) and let (o) be the Center of a Circle in an oblique View equal to it, at a Distance Oo in the same Line; with the Radius IO describe an Arch OP, which will denote the Section of Part of the *optical Plane*, for such a Plane must be a *Cylinder*. Then draw any Right Line GP parallel to OF, and it will be the Section of a *perspective Plane*, which is always a *plain Surface*. Join Io, and at right Angles thereto, draw the Diameter DE; lastly, draw the Lines AI, CI; aI, cI; also DI crossing OF in M; and IE, cutting OF in F, when produced.

Now the Optic Angle AIC subtended by the Circle in the *direct View* is to the Angle DIE which it subtends in the *oblique View*, as Io to IO, or inversely as their Distances from the Eye I, as is evident from the PRINCIPLES of OPTICS; therefore the Appearance of the Diameter DE upon the *optic Plane* will be LP which is

H

less

less than DE (= AC) in the same Ratio of IO to Io. From whence it appears, *that all Objects lessen in Appearance upon an OPTIC PLANE, in Proportion as their Distance from the Eye increases.*

But with regard to the *perspective Plane* GH, just the reverse will appear; for this Plane will cut the visual Rays IA, IC in G and N; and the Visuals DI, EI, in K and H. Hence GN will be the Perspective of AB; and KH the Perspective of the equal Line DE in the oblique View; and since GH is parallel to AF, we have $KH : GN :: MF : AC (= ac)$ but MF is greater than *ac* or AC; *therefore KH the Perspective of the Diameter of the Circle in the oblique View is greater than GN the Perspective of the same Diameter in the direct View.*

Hence also it is evident that KH being the Perspective of the Diameter of a PILLAR upon the Circle in the side View at (o) and GN the Perspective of the Diameter of an equal PILLAR in the direct View at O, *the Perspective of these equal PILLARS will be unequal, and the Perspective of the PILLAR viewed obliquely is always greater than the perspective of the same PILLAR placed directly before the Eye.*

But PILASTERS, being considered as Planes, have but *two Dimensions*; and being placed in the Front Line AF, must have all their Bases AC, *ac*, in the same Line; and the Perspectives of these Lines will be GN, *kb*; and they will be equal to each other; for it is $GN : kb :: AC : ac$; but $AC = ac$, therefore $GN = kb$. *And therefore also Pilasters or Planes standing upon equal Bases AC, ac, any where in the front Line*
AF,

AF, and having their perspective Bases GN, *kh*, equal, will have their own Perspectives every where equal likewise.

And from hence it appears that all Plane Surfaces whatever placed in a front Wall, or Plane, will have in their Perspectives no Change of Figure at all, a Square, a Parallelogram, a Triangle, a Pentagon, a Circle, an Ellipsis, &c. will be all the same Figures on the perspective Plane, and perfectly similar to the Originals; and this will hold good in every Part of such a Plane in Front as well above and below the Horizon, as on each Side the Eye.

Hence it is that the Places of the Sun, Moon, &c. and of their Images seen by Reflection, are equidistant from the Horizon of the Picture as well as from the Horizon of the Place of the Observer. Hence likewise if a BIRD were to fly perpendicularly upwards or downwards, its perspective Appearance would be always the same.

I might further observe, that as the Eye moves farther from the Plane of the Picture, the Perspectives of Objects encrease, and diminish as the Eye approaches the said Plane. Also that if ABC (FIG. 12.) be a Globe in direct View, and MDE an equal Globe in a side View, but in the same front Line AF, then the Perspective View of the former will be a *Globe* or Sphere, but that of the latter will be a *prolate Spheroid*. And many other Things will easily be accounted for by Skill in this Science, which often seem mysterious to those who understand not Perspective.

F I N I S.

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